A Logical Analysis of Autosegmental Approaches to Root-and-Pattern Morphology in Arabic

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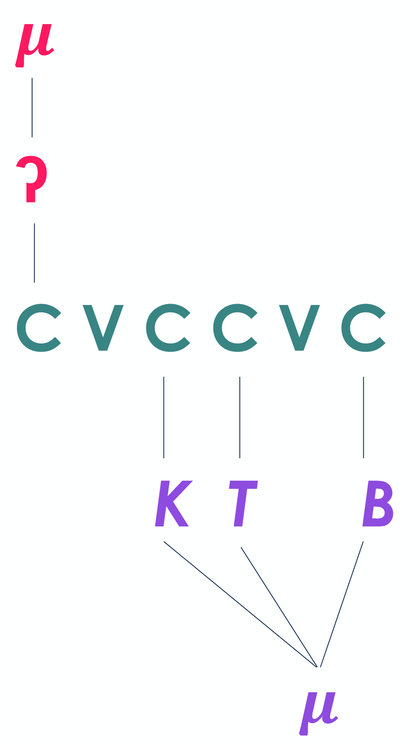
Overview

Autosegmental approaches to Arabic morphology generally take a **3-tier approach:**

- **Prosodic template**
- **Consonantal root**
- **Morphological affixes**

Example: **ʔaKTaB**



Association between tiers proceeds **left-to-right**

KTaaBaB

❌ Left-to-right association **exceeds regular computation** for autosegmental representations of **arbitrary length** (Jardine 2017)

Contribution: the **finite length** of the consonantal root in Arabic allows us to define:

- A logical **relational structure** (string model)
- **Logical transductions** (associations)

That yield well-formed Arabic autosegmental representations with only **First-Order Logic**.

Model Signature

Logical relational structure:

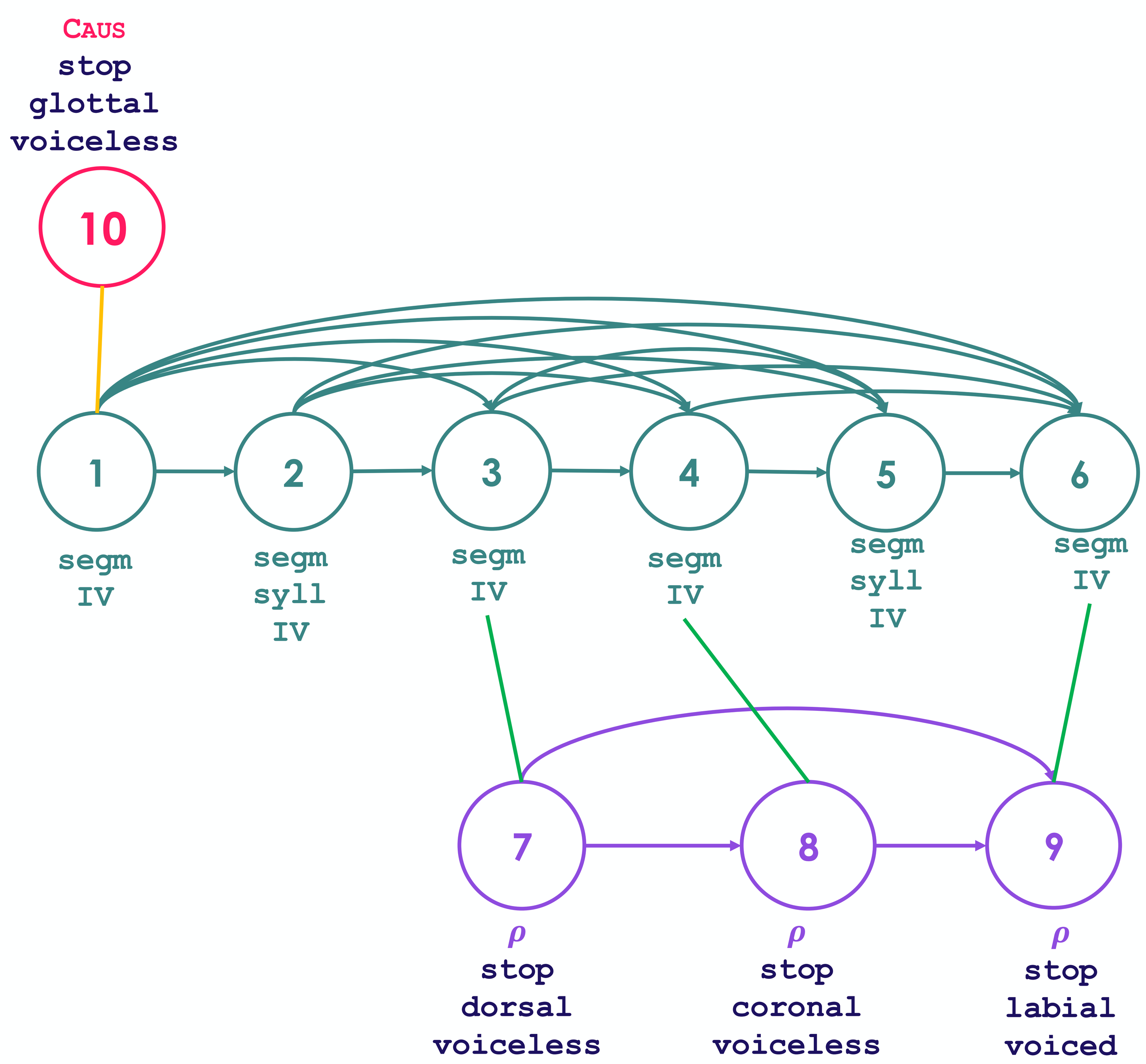
$$\mathfrak{R} = \{<, \circ_a, \circ_r\} \cup \mathcal{P} \cup \mathcal{C} \cup \mathcal{R} \cup \mathcal{M}$$

Binary relations:

- $<$ = general precedence
- $\circ_a$ = affix + prosodic template association
- $\circ_r$ = root + prosodic template association

Unary relations:

- $\mathcal{P}$ = prosodic template features: `segm`, `syll`, binyan
- $\mathcal{R}$ = consonantal root feature: ρ
- $\mathcal{M}$ = morphological affix features: `reflexive, etc.`
- $\mathcal{C}$ = phonological features for consonants



Affixation Transduction

We define the **consonantal "slots"** on our template that affixes could associate with:

$$\text{p\_slot}(x) = \bigvee_{p \in \mathcal{P}} p(x)$$

*Domain elements are **elements of the prosodic template** if they bear **some feature in** $\mathcal{P}$*

$$\text{c\_slot}(x) = \text{p\_slot}(x) \land \neg\text{syll}(x)$$

Consonantal slots** are **prosodic template elements** which **don't bear a `syll` feature

We then define a **successor ordering** over these "slots":

$$x \lhd_c y = \text{c\_slot}(x) \land \text{c\_slot}(y) \land x < y \land \nexists(z)[x < z < y \land \text{c\_slot}(z)]$$

successor consonantal slots** are elements which are **both consonantal slots** and **have no intermediate consonantal slot between them

and use this relation to define predicates for **absolute position:**

$$\text{first\_c}(x) = \text{c\_slot}(x) \land \nexists(y)[\text{c\_slot}(y) \land y \lhd_c x]$$

*The first **consonantal slot** is **not the successor of any other consonantal slot***

Finally, we use this to define a simple prefixation predicate:

$$\text{simple\_prefix}(x, y) = \text{first\_c}(y) \land \big(\text{IV}(y) \lor \text{V}(y) \lor \text{VI}(y) \lor \text{VII}(y)\big)$$

*Simple prefixation occurs to the **first consonant** in **Binyanim IV, V, VI, and VII***

See the full paper for the remaining Binyanim.

Root Association Transduction

We define an **unassociated predicate** to determine **which consonantal "slots" will be available for root association**, and define **successor ordering** over these:

$$x \lhd_{UC} y = \text{unassociated}(x) \land \text{unassociated}(y) \land x < y \land \nexists(z)[x < z < y \land \text{unassociated}(z)]$$

Successor unassociated consonantal slots** are elements that **are both unassociated** and **have no intermediate unassociated consonantal slot between them

We can use this to define the **first unassociated slot** like above:

$$\text{first\_UC}(x) = \text{unassociated}(x) \land \nexists(y)[y \lhd_{UC} x \land \text{unassociated}(y)]$$

*The **first unassociated slot** is **not the successor of any other unassociated slot***

And we can define the **first element in the root** similarly:

$$\text{first\_r}(x) = \text{root}(x) \land \nexists(z)[\text{root}(z) \land z < x]$$

The first root element** is **not the successor of any other root element

Because there are finitely many UCs and root positions, **we can define predicates for all positions in each one.**

Then the left-to-right association is:

$$\text{assoc}(x, y) = \big(\text{first\_UC}(y) \land \text{first\_r}(x)\big) \lor \big(\text{second\_UC}(y) \land \text{second\_r}(x)\big) \lor \big(\text{third\_UC}(y) \land \text{third\_r}(x)\big) \lor \big(\text{fourth\_UC}(y) \land \text{third\_r}(x)\big)$$

*We associate a root element and unassociated pattern slot if they are **both the first**, **both the second**, **both the third**, **or if the root is the third and the pattern is the fourth***

See the full paper for exceptions to this association.

Discussion & Future Work

Current work: complete account of autosegmental association for **triconsonantal roots** using **First-Order logic**

- But Arabic and Hebrew have **roots with more than 3 consonants!**

Key insight the same: the *finite* nature of the roots is what allows us to define these transductions

- As long as there is a **strict upper bound on the number of consonants in the root**, we should be able to obtain this result!
- **It seems there is!**

This analysis demonstrates **a computational advantage** of the **constrained set of possible lengths for Semitic roots.**

Citations: **Adam Jardine. 2017.** On the logical complexity of autosegmental representations. In *Proceedings of the 15th Meeting on the Mathematics of Language*, pages 22–35, London, UK. Association for Computational Linguistics.
John J McCarthy. 1981. A prosodic theory of nonconcatenative morphology. *Linguistic inquiry*, 12(3):373–418.

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