Marginal Sequences as a Window into Phonotactic Learning

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Attestation vs. Licitness

- Subcomponent attestation is closely related to licitness
- Subcomponents: syllable-based or linear k-factors
 - Toy example: given [can] and [dab], is [cab] acceptable?

Syllable sub-components

- [can] \Rightarrow [c] is a licit onset
- [dab] ⇒ [ab] is a licit rime
- [cab] = [c] + [ab]

Linear k-factors

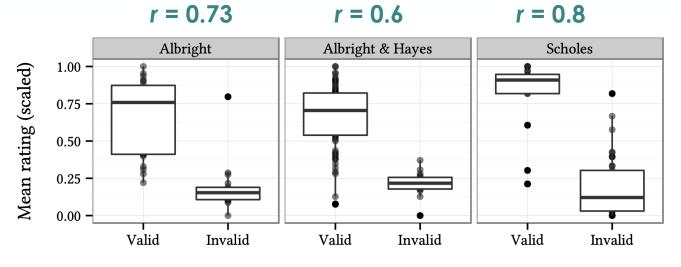
- 2-factors of [can] = {#c, ca, an, n#}
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- 2-factors of [cab] = {#c, ca, ab, b#}

Attestation vs. Licitness

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Gorman (2013)

Syllable-based attestation vs. English nonce-word judgments



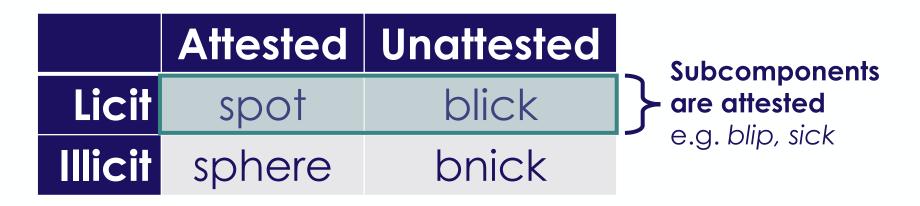
Kostyszyn & Heinz (2022)

2-factor attestation for Polish word-initial complex onsets

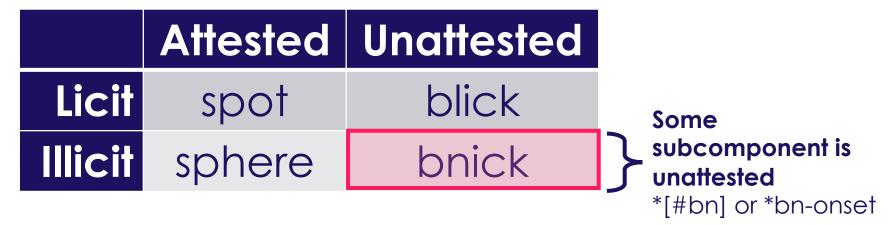
Pearson's **r** = 0.73

What's the causal relationship between attestation and licitness?

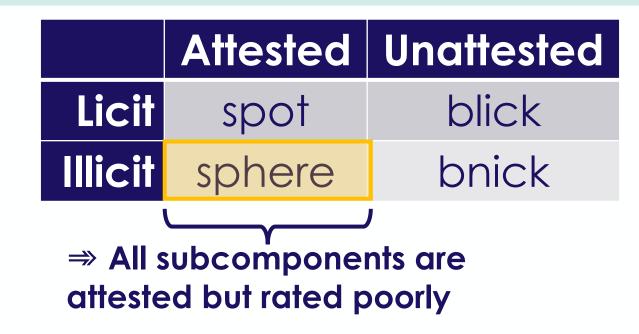
Gross phonotactic status



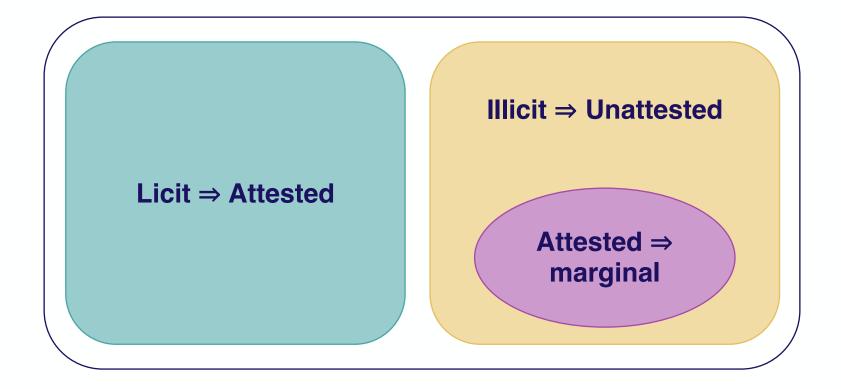
Traditional view: licitness \Rightarrow subcomponents are attested



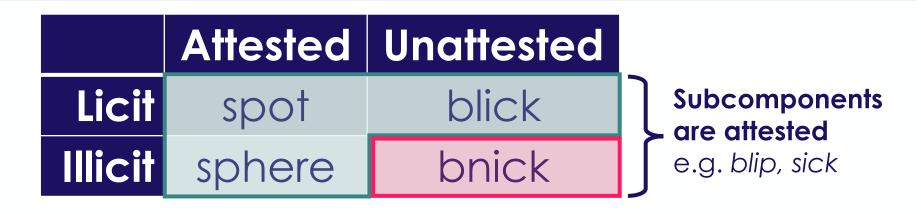
Traditional view: illicit ⇒ unattested subcomponent



Traditional view: marginal = exceptional subclass of illicit → Illicit but contain no unattested subcomponent

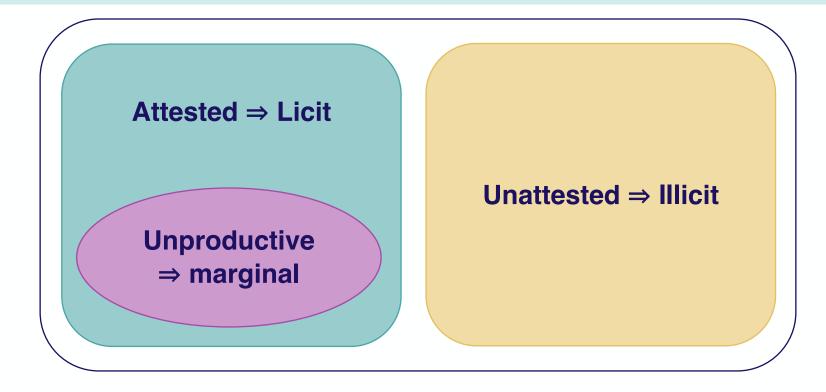


Attestation vs. Licitness Revisited



Subcomponents attested ⇒ licit Unattested subcomponent ⇒ illicit Marginal = exceptional subclass of attested Subcomponents attested but not licit

Attestation vs. Licitness: Proposal



The phonotactic grammar is **positive**, **syllable-based**, and **categorical**, with forms being either **licit**, **marginal**, or **illicit**

Outline

• Re-thinking the phonotactic grammar

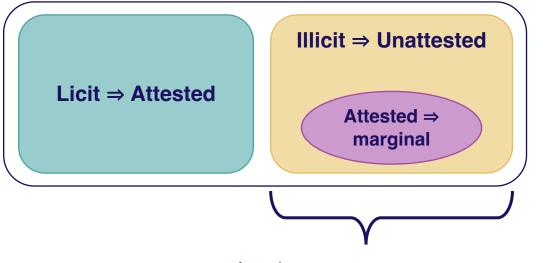
- Motivating observations
- What's (not) in the phonotactic grammar
- Phonotactic knowledge is non-linear
- A positive phonotactic grammar
- Phonotactic representations may be categorical
- Working Proposal
 - Proposal: Sequence-Wise Generalization Learner
 - Evaluation: English complex onsets
- Future work

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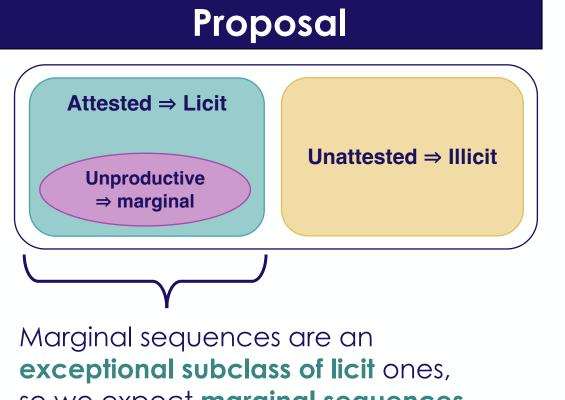
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Difference in Predictions

Traditional View



Marginal sequences are an exceptional subclass of illicit ones, so we expect marginal sequences to pattern like illicit ones



so we expect marginal sequences to pattern like licit ones

Evidence: Borrowings and Repairs

• Illicit forms are repaired in borrowings:

- Greek /pneumon/ → English /njumonia/
- German /pfitse/ → English /faiz./
- Spanish & Japanese: */#sC/

	Spanish	Japanese
Italian: /spagetti/	/espageti/	/supagetti/
Greek: /sfiŋks/	/esfinxe/	/swфinkwsw/
Greek: /sfaira/	/esfera/	(suupia)

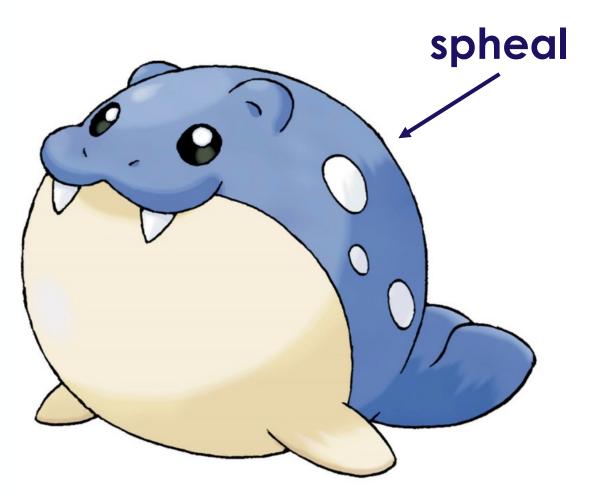
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Greek: /sfaira/	/esfera/	(suupia)	/sfij/

Evidence: New Words

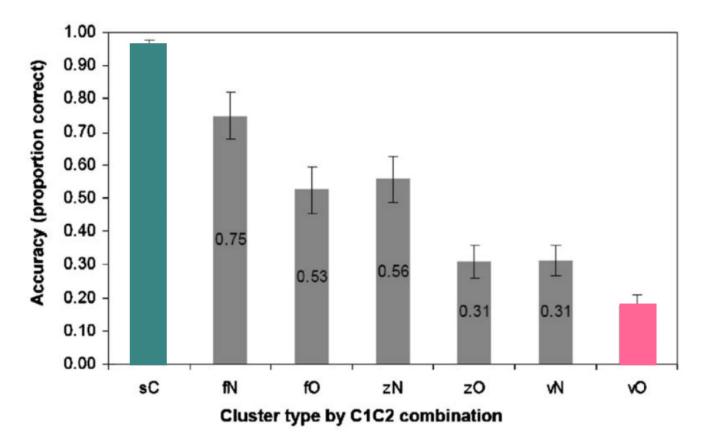


Evidence: New Words



Evidence: Production and Perception

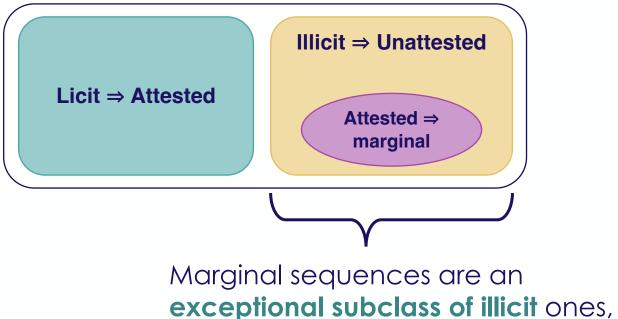
- Speakers have trouble producing illicit sequences
- But they don't have trouble producing /#sf/!
 - 97% accuracy /#sC/ sequences where
 C ∈ {f, p, t, k, m, n}



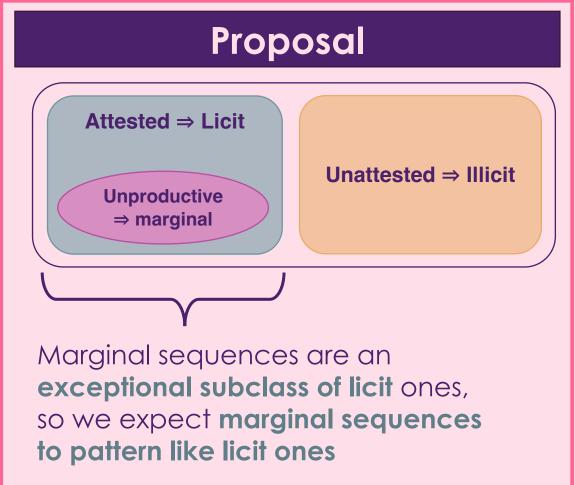
Davidson (2006)

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Any surface-true generalization that holds based on statistical inference over the lexicon, not necessarily resulting from phonological alternations or restrictions on the prosodic system Hayes & Wilson (2008)

MAXIMAL

MINIMAL

Any surface-true generalization that holds based on attestation in the lexicon, not necessarily resulting from phonological alternations or restrictions on the prosodic system, but restricted to certain computational classes Heinz (2010), Chandlee et al. (2019), Rawski (2021)

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MaxEnt and SEL: A Closer Look

Maximum Entropy

(Hayes & Wilson 2008)

- Negative grammar of markedness constraints
- Weighted markedness constraints ⇒ probability of output
- Goal of learning = determine constraints and ranking that maximize probability of observed forms
- Guaranteed to find global maximum

String Extension Learning

(Heinz 2010)

- Positive grammar of k-factors
- Accumulate k-factors from the input
 - k-factors = subcomponents of length k
- Add k-factors to the grammar as they are seen
- A string is licit if **all of its k**factors are licit
- Learnable in the Limit from Positive Data

MaxEnt and SEL: Handling Marginal Forms

Maximum Entropy

- Weight e.g. *[#sf] less than
 *[#bn]
 - Violating *[#sf] is less bad
- Hayes & Wilson remove "exotic onsets" from train
 - Performance hit when they're included

String Extension Learning

- If all k-factors seen in input, then string is licit
- No distinction between marginal and licit forms

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No static phonotactics: the phonotactic grammar contains only generalizations that result from phonological alternations or restrictions on the prosodic inventory Gorman (2013)

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Morpheme structure constraints: the phonotactic grammar contains only generalizations that result from restrictions on the prosodic inventory of underlying **representations** Chomsky & Halle (1968)

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Comon 2013

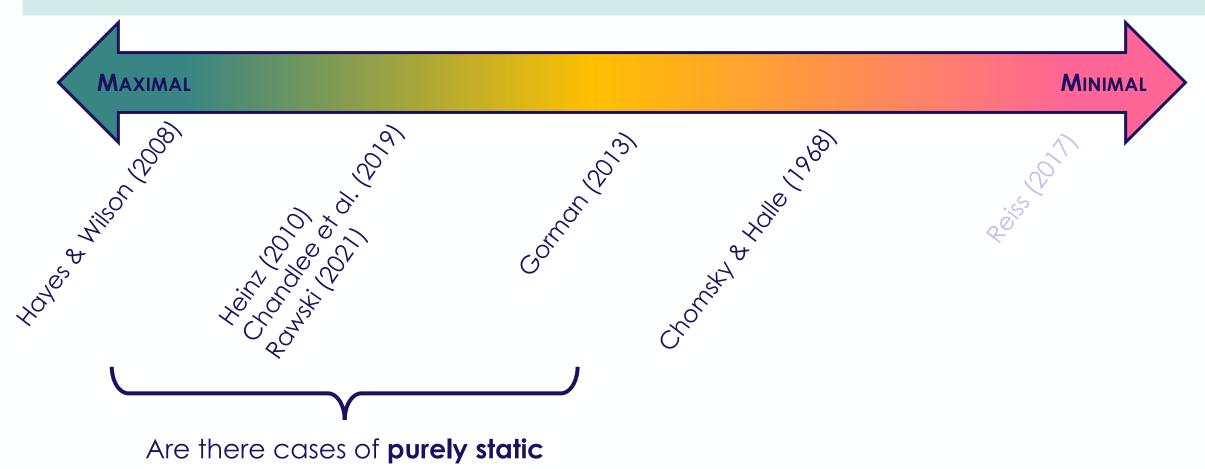
Leiner Construction

Cronst + tolle 1968 No phonotactic grammar: Phonotactic generalizations play no part in the phonological grammar but are rather emergent, metalinguistic knowledge **Reiss (2017)**

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HOVES WILSON 2008



restrictions that are synchronically active in speakers' grammars?

Inactive Static Restrictions: Turkish

- BACKNESS HARMONY
 - 61% of roots conform
- **ROUNDNESS HARMONY**
 - Applies to high vowels
 - 83% of roots conform

• LABIAL ATTRACTION

- High back vowels tend to be round after alabial consonant sequences
- Not reflected in alternations
- 69% of roots conform

Nom.Sg	Nom.PL	Dat.Sg
pelür	pelür ler	pelür ü
boğaz	boğaz la r	boğaz ı
ip	ip ler	ipi

Inactive Static Restrictions: Turkish

- Zimmer (1969) paired word-likeness task: which is better?
- Goodman-Kruskall **y** measured for each restriction:
 - BACKNESS HARMONY: γ = 0.694 🔽 🧎
 - Roundess Harmony: $\gamma = 0.68$
 - LABIAL ATTRACTION: $\gamma = -0.043 \times$ Purely static
- Suggests a more **minimal** view of the phonotactic grammar
 - Not all surface-true generalizations will be grammaticalized
 - Current work: focus on **restrictions on prosodic inventory**

Reflected in alternations

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Subcomponents Revisited

Syllable sub-components

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- [dab] ⇒ [ab] is a licit rime
- [cab] = [c] + [ab]

Linear k-factors

- 2-factors of [can] = {#c, ca, an, n#}
- 2-factors of [dab] = {#d, da, ab, b#}
- 2-factors of [cab] = {#c, ca, ab, b#}
 Hayes & Wilson (2008) Heinz (2010) Chandlee et al. (2019) Rawski (2021)

Equivalence?

- Linear representations can be augmented with syllable boundaries
 - 2-factors of [hæ.pi] = {#h, hæ, æ., .p, pi, i#}
- Inherent generalization power is still different:

Linear + Syllable Boundaries

- Will need **k > 3** to capture clusters
- [d.n] and [b.m] in the observed k-factors

Syllable-Based Representations

- Observe [d] and [b] as licit codas
- Observe [m] and [n] as licit onsets
- [d.m] and [b.n] will be accepted
- [d.m] and [b.n] will not be accepted

Which do humans do?

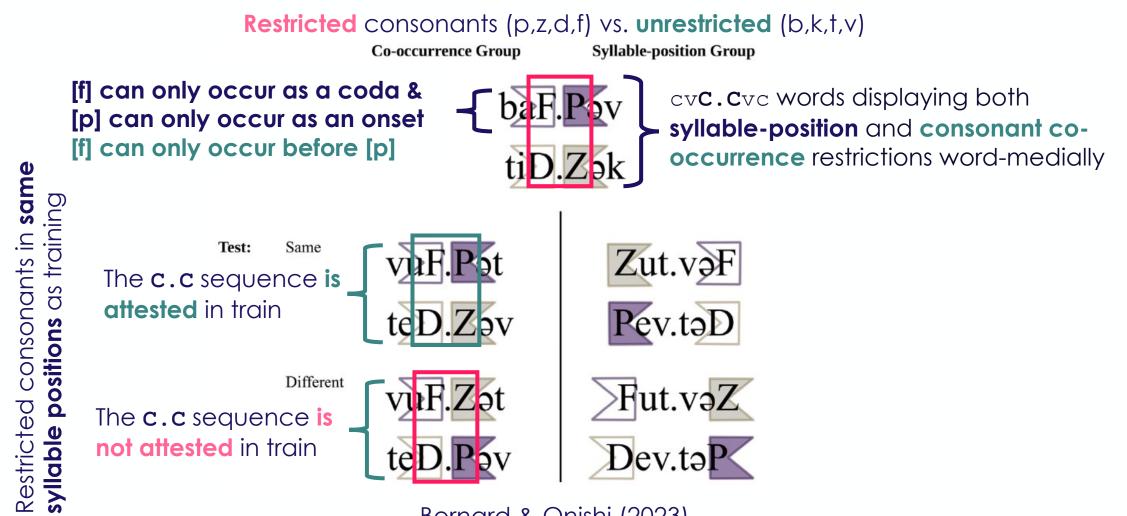
Evidence for Non-Linear Representations

- Bernard & Onishi (2023): infants & children spontaneously represent phonotactic restrictions over syllables
- Kabak & Idsardi (2007): adult Korean speaker's illusory vowel perception is governed by syllable-position restrictions
- Extremely **early sensitivity** to syllables

Evidence for Non-Linear Representations

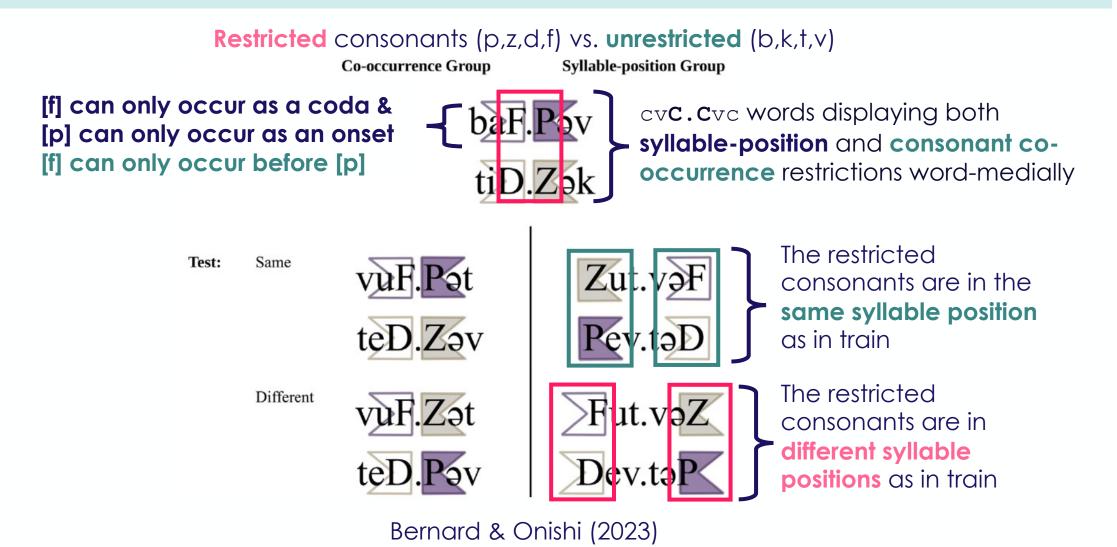
- Bernard & Onishi (2023): infants & children spontaneously represent phonotactic restrictions over syllables
 - Children (5;0) and infants (0;11) (55.5-65.8 months) (10.6-12.1 months)
 - Distinguish sensitivity to linear co-occurrence vs. syllable position

Spontaneous Representation of Restrictions



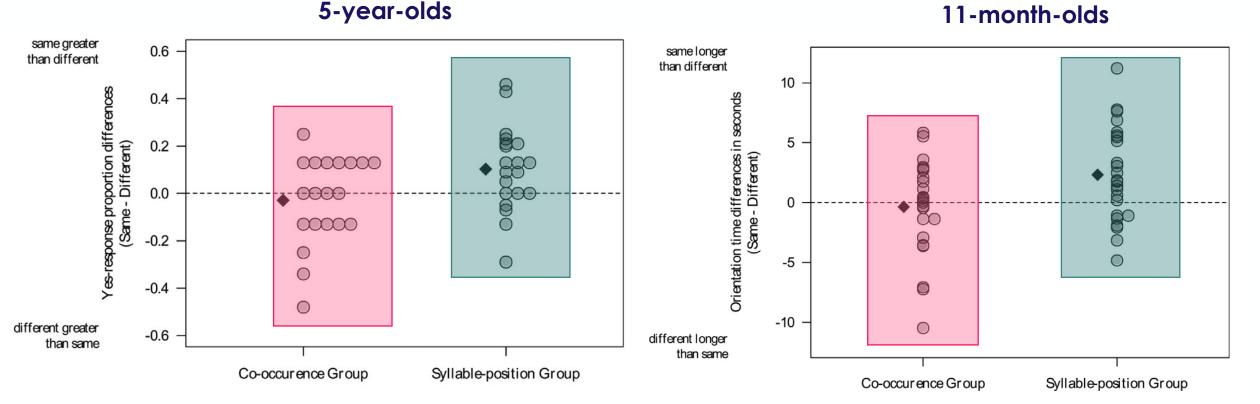
Bernard & Onishi (2023) Payne: Marginal Sequences & Phonotactic Learning

Spontaneous Representation of Restrictions



Payne: Marginal Sequences & Phonotactic Learning

Spontaneous Representation of Restrictions



Children & infants exploit syllable structure in phonotactic learning even when other information is available

Bernard & Onishi (2023)

Payne: Marginal Sequences & Phonotactic Learning

Evidence for Non-Linear Representations

- Bernard & Onishi (2023): infants & children spontaneously represent phonotactic restrictions over syllables
- Kabak & Idsardi (2007): adult Korean speaker's illusory vowel perception is governed by syllable-position restrictions
 - $VC_1 . C_2 V$ sequences are generally ok in Korean but some unattested
 - Contact: C_1 is a licit coda and C_2 is a licit onset, but $C_1 \cdot C_2$ unattested
 - ***[k.m]** because **[k]** undergoes nasalization to **[ŋ.m]**
 - Syllable-position: C₁ is unattested as coda or C₂ unattested as onset
 - *[c.] *[r.] for codas and *[.l] *[.ŋ] for onsets
 - Korean-speaking adults can discriminate $vc_1 . c_2 v$ from $v.c_1 v.c_2 v$ in the **contact** case but struggle in the **syllable-position** case
 - Syllable-based account predicts this asymmetry

Evidence for Non-Linear Representations

- Bernard & Onishi (2023): infants & children spontaneously represent phonotactic restrictions over syllables
- Kabak & Idsardi (2007): adult Korean speaker's illusory vowel perception is governed by syllable-position restrictions
- Extremely early sensitivity to syllables
 - **Bijeljac-Babic et al (1993): 4-day-old infants** discriminate words based on number of syllables but not number of phonemes
 - Bertocini & Mehler (1981): infants can discriminate syllable-like stimuli better than non-syllable stimuli before 0;2
 - Peters (1983): word segmentation errors align with syllable boundaries

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Positive & Negative Grammars: Equivalence

- Dominant View: phonotactic grammar made up of negative constraints (e.g., *[#bn]) (Prince & Smolensky 1993, Hayes & Wilson 2008, Dai 2024, i.a.)
- Why not store sub-components that are allowed?
- Model Theory tells us:
 - Over segments: straightforward conversion between grammar types
 - Over feature bundles: the same algorithm can be used to learn both
- From a computational perspective, **no a-priori reason to favor a negative grammar**

Positive & Negative Grammars: Equivalence

• Model Theory tells us:

- Over segments: straightforward conversion between grammar types
 - Toy example: 2-factor grammar, $\Sigma = \{V, C\}$
 - Positive grammar:
 G⁺ = {VC, CV}
 - Negative grammar: $G^- = \Sigma^2 \setminus G^+ = \{VV, CC, CV, VC\} \setminus \{VC, CV\} = \{VV, CC\}$
 - Banning \mathbf{vv} and \mathbf{cc} or only allowing \mathbf{vc} and $\mathbf{cv} \Rightarrow$ same language!

Positive & Negative Grammars: Equivalence

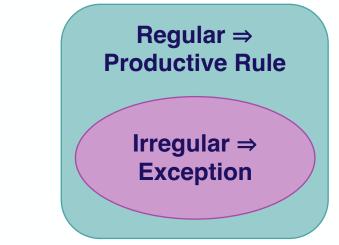
• Model Theory tells us:

- Over segments: straightforward conversion between grammar types
- Over feature bundles: the same algorithm can be used to learn both
 - Chandlee et al (2019) & Rawski (2021): algorithm to learn only negative grammars over sequences of feature bundles
 - *[+NAs][-Son, -Voi] instead of *nt, *mp, *nk, etc. separately
 - Prohibitively costly to convert between negative & positive grammars of feature bundles
 - Payne (2024): if we fix k (the size of the elements in the grammar), we can adapt this algorithm to learn positive and negative grammars with the same guarantees

Computationally, no advantage to a negative grammar

Is Phonotactic Learning Really so Different?

- Syntax: positive grammar Chomsky (1957, 1992); Liang et al. (2022); Li & Schuler (2023) i.a.
- Morphology: positive grammar Pinker (1998); Yang (2016); Belth et al. (2021) i.a.
- Phonology:
 - Rule-based view: **positive grammar** Chomsky & Halle (1968); Belth (2023, 2024), i.a.
 - Optimality Theory: negative grammar Prince & Smolensky (1993); McCarthy (2007, 2008), i.a.
- Phonotactics: is it different?



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Expand the learning approaches from these subfields to phonotactics

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Gradient Representations?

- Experimental studies on **phonotactic acceptability judgments** generally report **gradient results** (Scholes 1966, Frisch et al. 2000, Albright 2009, Daland et al. 2011, i.a.)
- **Dominant view:** gradient acceptability judgment results should be accounted for by a **gradient phonotactic grammar** (Albright 2009, Frisch et al. 2000, Hayes & Wilson 2008, Shademan 2006, Daland et al. 2011, i.a.)
 - Equate probabilistic likelihood with phonotactic well-formedness
- Gradience could also result from experimental methodology

Eliciting b	inary	judgr	ments	but
reporting	avera	iged	results	

Eliciting Likert-scale judgments

Averaged Binary Judgments

- Scholes (1966): "could this be a word of English?" (yes/no)
- Report number of participants who gave yes judgment
- Toy example: 8/10 participants give yes judgment

Gradient Interpretation:

The word is **80% acceptable** in **any given speaker's grammar**

Erases possibility of individual variation

Gradience in averaged binary judgments ⇒ gradience in phonotactic representations

Categorical Interpretation:

The word was **completely licit** for 8 speakers and **completely illicit** for 2

Individual variation causes gradience when averaged over speakers

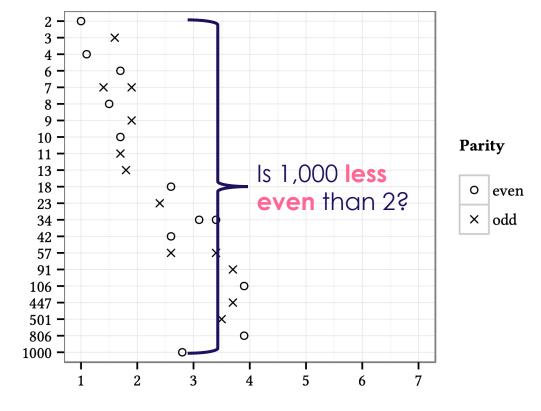
Unfortunately, by-speaker results not available for most studies

Likert-Scale Judgments

- Daland et al. (2011): "how likely is this word to become a word of English in the 21st century, on a scale of 1-6?"
- Report average rating of each word
- Likert scales are known to produce task effects

Likert-Scale Judgments

- Likert scales are known to produce task effects
- Armstrong, Gleitman & Gleitman (1983): how representative are numbers of even or odd?
 - Gorman (2013): similar task effect may occur for acceptability judgments
 - Schütze (2011): gradience may emerge when subjects try to reconcile categorical grammar with gradient task



Mean rating

Number

Likert-Scale Judgments

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Gradience in Likert scale judgments ≠> gradience in phonotactic representations

Gradient Representations?

Gradience in averaged binary judgments ⇒ gradience in phonotactic representations Gradience in Likert scale judgments ⇒ gradience in phonotactic representations

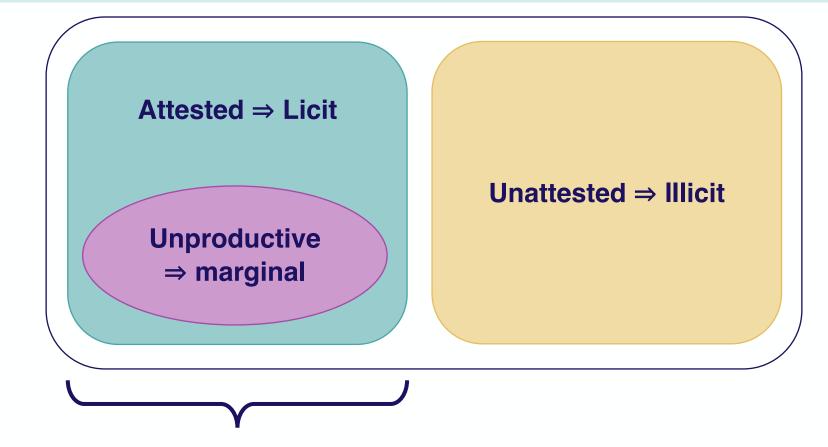
- Gradient judgments ⇒ gradient phonotactic grammar
- Possibility of task effects # categorical phonotactic grammar
- Some reasons to favor a categorical approach
 - We can successfully elicit categorical judgments
 - Binaries are simpler and don't require scalar computation
 - Other parts of the grammar (e.g., syntax) are generally considered categorical ⇒ internal consistency

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Attestation vs. Licitness: Proposal



How do we learn whether a subcomponent is licit or marginal?

Motivating Observations

Licit: [sp]-onset

- Occurs before a wide range of vowels
 - spat, spell, spot, sputter
- Belongs to [s]-[voicelessstop] onsets
 - [sp], [st], [sk] all licit

Marginal: [sf]-onset

- Occurs before a limited
 number of vowels
 - sphere, sphinx
- Only similar onset = [sv]
 - svelte also marginal

Working Proposal: "combinatorial power" of syllable subcomponents related to licitness

Proposal: Measuring Combinatorial Power

The Tolerance-Sufficiency Principle

- Threshold for generalization based on computational efficiency
 - Children will generalize a rule when it's more efficient to
- Given a rule *R* applicable to *N* types and seen applying to *M* of those types, **generalize the rule iff:**

$$N-M \leq \boldsymbol{\theta}_N = \frac{N}{\ln N}$$

Yang (2016)

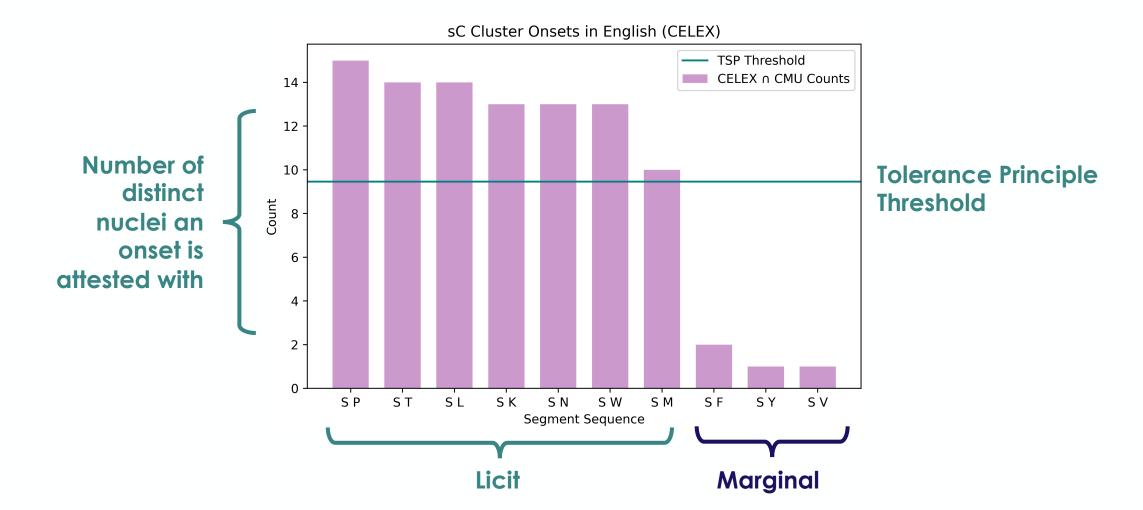
Proposal: Measuring Combinatorial Power

The Tolerance-Sufficiency Principle

 In a language with N possible nuclei, an attested onset/coda is licit iff it occurs with at least M of those nuclei and

$$N-M \le \boldsymbol{\theta}_N = \frac{N}{\ln N}$$

Illustration: English [sC] Onsets



Sequence-Wise Generalization Learner

- Recursive, feature-based subdivision to learn phonotactics as increasingly-specific sequences of feature sets
 - Parallel to Belth, Payne et al. (2021) for morphological learning
- At each step, intersect all subcomponents in the current input to give some **underspecified sequence S**
 - If sufficiently many syllable subcomponents matching S are licit, add S to the set of licit subcomponents
 - Otherwise, **subdivide the input** based on the most frequent feature set at the index in the string with greatest difference between **N** and **M**
- If no generalization & no more features to subdivide on, S is marginal

Proposal: Measuring Generalizability

- Given some S, are a sufficient number of subcomponents fitting it licit?
 - Let $N = \prod n_i$ where n_i = # segments that fit features at position i
 - Let *M* be the number of distinct syllable subcomponents observed that fit the entire feature set & are licit
 - Check if $M N \leq \frac{N}{\ln N}$

Proposal: Illustration

- Example: English complex onsets
 - $N([+SIBILIANT] [-SON, -CONT]) = |\{z, s\} \times \{p, t, k, b, d, g\}| = 12$
 - *M* = number of licit subcomponents that fit [+SIBILIANT] [-SON, -CONT]
 - {sp, st, sk} are licit $\Rightarrow M = 3$
 - $N M = 12 3 = 9 > \theta_{12} \approx 4.8 \times$
 - Subdivide: find position with greatest difference between number of observed & number of possible segments
 - First position: 2 possible, 1 observed \Rightarrow 1 difference
 - Second position: 6 possible, 3 observed \Rightarrow 3 difference
 - Add most frequent feature occurring at this position: $\pm Voice$
 - Recurse: [+SIBILIANT] [-SON, -CONT, -VOI] vs. [+SIBILIANT] [-SON, -CONT, +VOI]

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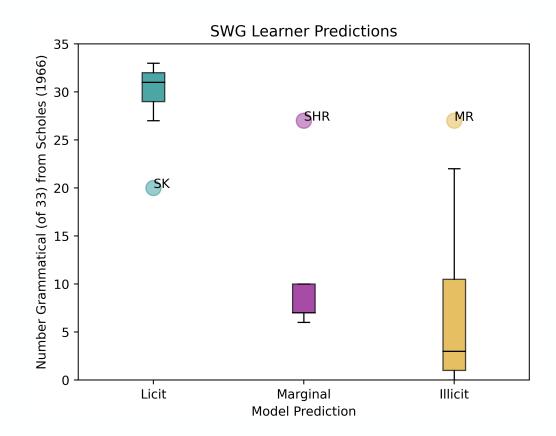
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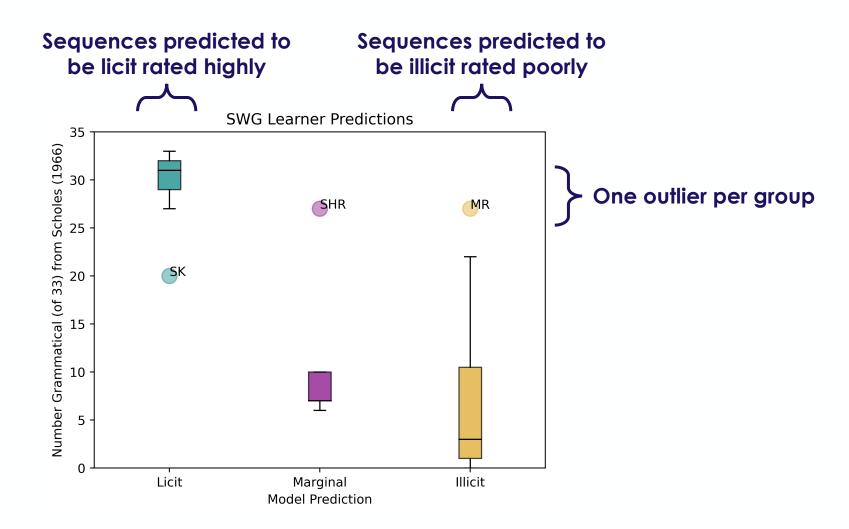
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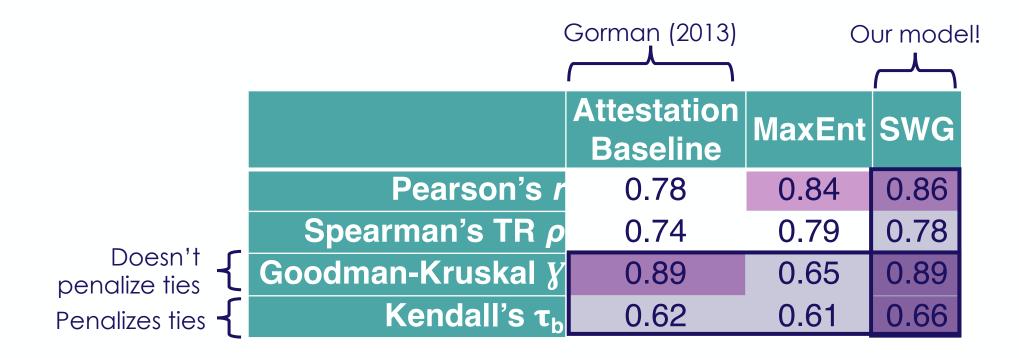
- Apply the model to real data: English complex onsets
 - CELEX \cap CMU: ~41k words
 - Transcribed using the CMU Pronouncing Dictionary
 - Syllabified using the tool from Gorman (2013)
 - Distinctive features encoded for ARPABET based on those in Hayes & Wilson (2008)
 - Features can be **positive**, **negative**, **or unspecified**

Scholes (1966): complex onsets in monosyllabic nonce words

• Binary decisions by 33 seventh graders







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Future work

Testing Model Predictions

Model predictions

- Initial stage of conservatism
- Accumulate sufficient evidence

Further testing & comparison

- Polish complex onsets
- More judgments (e.g., Daland et al. 2011)
- Comparison with more **other models**
- Experimental investigation
 - Languages with smaller vowel spaces
 - Artificial language studies

Features or Segments?

- Some evidence for early underspecificaition
 - English-learning children cannot discriminate /bɪ/ and /dɪ/ when lexical contrast is implicated but can discriminate [b] and [d] when phonetic contrast is implicated (Stager & Werker 1997)
 - French-learning 11-month-olds do not prefer known words to alternates with different voicing or manner (Hallé & Boysson-Bardies 1996)
- In practice, recursion almost always leads to maximallyspecified feature set sequences
 - No measurable differences between segments & features in terms of correlation with human judgments on full training
- Is phonotactic knowledge underspecified?

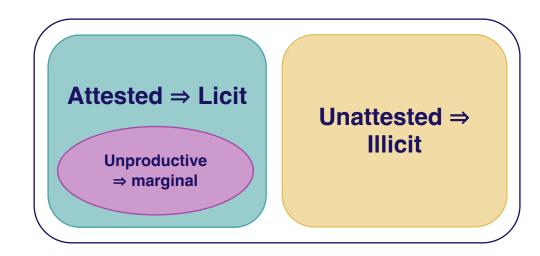
Features or Segments?

• Can we make **phonotactic generalizations** based on features?

- sp, st \rightarrow sk \checkmark
- sm, sn \rightarrow sŋ \times
- Is there something **special about [ŋ]** or is what's allowed/disallowed **too arbitrary** to allow for feature-based generalization?

Conclusions

- The phonotactic grammar is:
 - Positive
 - Categorical
 - Syllable-based
 - Minimal: contains no static restrictions
- Preliminary learning model in this framework
 - Uses recursive search with the Tolerance-Sufficiency Principle
 - Categorizes attested subcomponents as licit or marginal
 - Matches better with the judgments of **Scholes et al.** than MaxEnt



Thank you!!

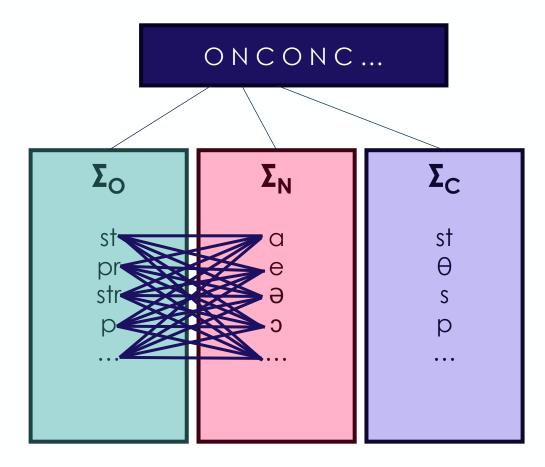
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Extra Slides

Linear vs. Syllable-Based Representations



- We can think of the syllable-based representation being **SL over 3** alphabets
- Can convert this to a **single**, **linear SL grammar** straightforwardly
 - For each transition, **add all possible combinations** except those that are disallowed (i.e. marginal)
 - The grammars will generate the same language but the linear one doesn't build in generalization

Previous Work: Gradient Models

- MaxEnt (Hayes & Wilson 2008): well-formedness = probability
 - Weighted markedness constraints \Rightarrow probability of output
 - Goal of learning = determine constraints and ranking that maximize probability of observed forms
 - Guaranteed to find global maximum

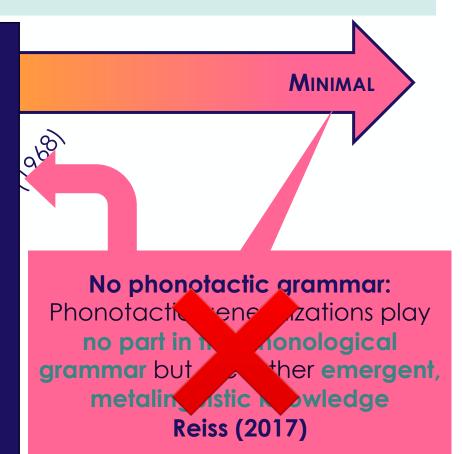
Previous Work: Categorical Models

- String-Extension Learning (SEL, Heinz 2010): accumulate k-factors from the input to form a positive grammar
 - Initial grammar = ϕ
 - For some input t[i], the output of the learner ϕ is: $\phi(t[i]) = \phi(t[i-1]) \cup \{x \in \Sigma^k : \exists u, v \in \Sigma^*, w = uxv\}$
 - The language of the resulting grammar is given by: $L(G) = \{ w \in \Sigma^* : fac_k(w) \subseteq G \}$
 - Strictly Local languages are Learnable in the Limit from Positive Data

What's in the Phonotactic Grammar?

3 Arguments:

- C dient judgments incompatible with categorical grammar Task effects are possible!
- Planotactic judgments are **colored** by orthography, Iternations, experience with other languages, etc.
 - Is this not true of all linguistic judgments?
 - Djects have accurate judgments for languages they don't now
 - [pumehana] vs. [bɛzvzglɛndni]: Polish vs. Hawaiian
 - Just need to know Hawaiian doesn't allow CC
 - What about more nuanced judgments: [sfin] vs. [stin]?



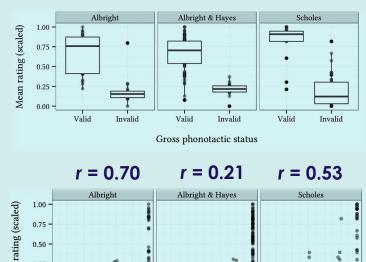
Gradient vs. Categorical: Previous Work

Gorman (2013)

Onsets & rimes are well-formed if they appear in a representative sample

r = 0.73

r = 0.6r = 0.8



MaxEnt probability (log)

Durvasula (2020)

Attestation-based categorical baselines perform at least as well as MaxEnt

When applied to the Scholes (1966) judgment data, type frequency of the onset sequence does not affect model fit, raising questions about where gradience in acceptability comes from.

Kostyszyn & Heinz (2020)

2-factor attestation for Polish word-initial complex onsets predicts acceptability better than the MaxEnt model:

2-factor Pearson's r = 0.73

MaxEnt Pearson's r = -0.07

0.50

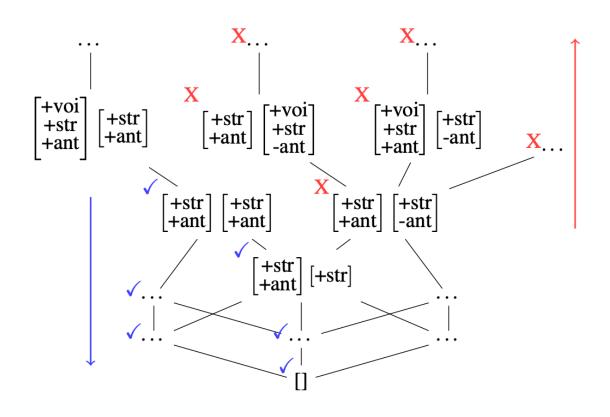
Mean 0.25

The Cost of Underspecification

- Far more possible *k*-factors when we allow for underspecification
 - Model with *n* binary features: s < 2ⁿ segments
 - s^k < (2n)^k possible k-factors
 - Underspecification ⇒ ternary features: (3n)^k possible k-factors
- Interdefinition algorithm less straightforward:
 - To determine if a *k*-subfactor **should be added to G+**:
 - Check if it's in G-
 - Also check if any of its sub-factors or super-factors are in G-

Positive & Negative Grammars: BUFIA

- Chandlee et al. (2019) & Rawski (2021):
 - Traversal that exploits partially ordered hypothesis space
 - Only continue to search if som k-factor matching the description is attested
 - Otherwise, learn constraint
- Constraints of length $\leq k$



Positive & Negative Grammars: BUFIA

- Payne (2024): positive grammars require all factors be of exactly size k in order to tile
 - Extend BUFIA to learn both positive & negative grammars:
 - A factor is **allowed** if:
 - All sequences matching it are **attested**
 - None of the sequences matching it are **unattested**
 - A factor is banned if:
 - All sequences matching it are **unattested**
 - None of the sequences matching it are **attested**
 - Same learning guarantees as BUFIA!

	$\in G$	$ \not\in G $
A	Positive Grammar (Equation 13)	Negative Grammar (Equation 10)
Ξ	Negative Grammar (Equation 11)	Positive Grammar (Equation 14)

 $-\alpha$

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