Formally defining the learning setting for child language **Jordan Kodner** Sarah Payne acquisition **Stony Brook University** The 101st LSA

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Computational Learning Theory (CLT)

- A subfield of computational theory and AI
- Developed as the formal side of machine learning, but formally describes *all* learning, including by biological systems

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Cognitive science combines these fields.

Can CLA and CLT be unified to elucidate linguistic cognition?



CLA × CLT: Why Combine Them?

From the perspective of Child Language Acquisition

- Reveal connections to other kinds of learning using shared formalisms
- Uncover explanatory gaps and new research pathways

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- Reveal connections to other kinds of learning using shared formalisms
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From the perspective of Computational Learning Theory

- Direct research effort at the heart of the problem of human language learning
- Offer a means to incorporate more empirical information into modeling

Easier said than done!

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→ Well-described CLT frameworks do not neatly apply out-of-the-box

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Today's goal:

We'll lay out some desiderata for a formalization of CLA

↔ CLA-able framework for CLT

What should be formally defined for CLA?

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In terms more familiar to CLA and theoretical linguists:

• What does the input data look like?

In terms of representations and distributions

 What does the output of learning look like? In terms of representation (the grammar) and how it manifests in behavior In terms of intermediate as well as final learner states

Desiderata: The Input

Language is the primary form of input to CLA

- Certainly there is other input as well (multi-sensory, world grounding...)
- But non-linguistic input is often apparently not necessary or even helpful Blind children follow nearly identical learning trajectories to seeing children¹ Blind adults demonstrate near-identical semantics for sight words² NLP/cog modeling systems don't necessarily benefit from multimodal input³

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"The reports of my death are greatly exaggerated"

- The Primacy of Language Input

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Implications for CLT

(Except when demonstrated otherwise) language-only input is appropriate

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Implications for CLT

- (Except when demonstrated otherwise) language-only input is appropriate
- Learning-focused math ling research is already on track for CLA 😀

"The Poverty of the Stimulus"

Small, sparse, skewed input is a quantitative fact of CLA

A particular challenge for any kind of naturalistic language learning
 → the argument from the Poverty of the Stimulus (Input Sparsity Problem)

More generally, for our purposes today:

- **X** Simple, brute force, *tabula rasa* learning strategies will exhaust the input before successful learning
- Carefully selected representations + clever hypothesis generation
- or hypothesis search is necessary
- This is the kind of stuff CLT is great at!

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 → the perpetual search for more, more, more training data
 But children can't just add more training data...

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Consequence:

A CLA-relevant CLT framework should incorporate the Input Sparsity Problem

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Long-Tailed Distributions in Child-Directed Speech

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And of derived distributions:

Morphological paradigm saturation, lexical attestation in constructions...

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- They also apply outside of language: SEARS Catalogue¹

Number of items of like price

Number of items with a similar number of styles

Number of products per page

Number of pictures per product

¹Zipf 1950, ²Adamatzky 2022, ³Newitz 2013, ⁴Sorbaro et al. 2019, ⁵Lin 2009

...

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Electrical Spiking among Fungi² 🍄 🗲 Intervals between spikes

Average spike amplitude Number of spikes in trains

...

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- They also apply outside of language¹⁻⁵

Consequence: Most input instances are ultimately redundant, uninformative, maybe even distracting?But at least they aren't deliberately adversarial

CLA "ends" eventually

- The Critical Period¹: the grammar is acquired during CLA then remains nearly fixed
- There is no "cliff." CLA trails off over time. We still learn certain aspects of language, like vocabulary, well in adulthood
- Most aspects of the grammar are acquired well before the end of the period



Age of Acquisition of a Second Language

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Age of Acquisition of a Second Language

Language Exhibits a "Critical Period"

- There is no "cliff." CLA trails off over time. We still learn certain aspects of language, like vocabulary, well in adulthood
- Most aspects of the grammar are acquired well before the end of the period

Consequences:

The input is not only finite but "small" by modern NLP standards. Pieces of the grammar are acquired on different "small" input sizes

- Learners receive on the order of 10 million tokens per year¹
- Individual learner vocabularies grow over the course of development²



¹Gilkerson et al. 2017, ²Fenson et al 1994, Hart & Risley 2003, Plots from Fenson et al 1994

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Our attempt to rescale these plots to match (the 90th percentile lines are inconsistent)



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Measuring lexical overlap with Jaccard similarity [0-none, 1-complete] The four children's subcorpora in the Providence Corpus range from 0.25 (Naima-William) to 0.37 (Ethan-William)¹

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Comparable range to the Brown, Brent, and MacWhinney CDS corpora vs the ~1000 most frequent words in COCA genre corpora 0.21 (Academic'92-Brent) to 0.44 (Fiction'04-MacWhinney)²

Children do not receive the same input instances

- Not even children in the same environment receive identical input^{1,2}
- Though given input skew, high frequency instances are more likely to appear to more children and are more likely to appear early in the input sequence
- Despite all this, children exhibit stark uniformity in learning outcomes³

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Consequence:

CLA learners cannot generally assume specific instances will be present in the input, even though they can assume distributions. Outcomes should be robust to variation in the input

Direct Negative Feedback

- Sparse and not consistently provided
- It's noisy ever misunderstood a toddler?
- Famously ignored/misunderstood by children¹

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Child: Nobody don't like me. McNeill (1966) Mother: No, say "nobody likes me." Child: Nobody don't like me. [Eight repetitions of this dialogue follow.] Mother: No, now listen carefully, say "NOBODY LIKES ME." Child: Oh! Nobody don't likes me.

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Child: My teacher holded the baby rabbits and we patted them.
Adult: Did you say your teacher held the baby rabbits?
Child: Yes.
Adult: What did you say she did?
Child: She holded the baby rabbits and we patted them.
Adult: Did you say she held them tightly?
Child: No, she holded them loosely. Cazden (1972)

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Indirect Negative Feedback

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Consequence: CLA must proceed from positive evidence only

The Input to CLA: Summary

The input is small, sparse, and skewed

- Pieces of the grammar are acquired at different times
 → Specific problems call for specific input sizes (not learning in the limit)
- Long-tailed distributions are ubiquitous
 Lots of redundant data. Informative inputs may be few and far between
- Specific input instances are highly variable between learners
 → CLA requires some substantial degree of robustness
- Negative evidence is hard to come by, unreliable, and ignored by learners

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The big picture takeaway: Child language acquisition is fundamentally a game of generalization from positive examples

The Input to CLA: Summary

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•	Pieces o	f the grammar are acquired at different times	
	→ Specif	The Big Caveat:	g in the limit)
•	Long-tai	None of this is to say that writing proofs about	
	→ Lots o	learning in the limit, guarantees about learning under	between
•	Specific	any arbitrary distribution, etc., isn't worthwhile for	
	→ CLA re	linguists. Far from it.	
•	Negative	It just isn't strictly CLA × CLT	learners

The big picture takeaway: Child language acquisition is fundamentally a game of generalization from positive examples

Desiderata: The Output

CLA takes in language input and outputs a grammar. More formally,

$h: L(G) \rightarrow G$

Language acquisition is a function that takes

some language generated by a grammar as input and yields some grammar as output

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L(G) is what we've been talking about as "the input"

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$h(L(g_1)) = g_2, g_1, g_2 \in G$

A learner whose input was generated by a grammar g_1 should acquire grammar g_2

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- This characterization may be obvious, but it leaves far too much unsaid
- As we all know: the nature of *h* and *G* really matters
- Less thought about: So does relationship between g₁ and g₂!

What is G?: A shared interest

- Theoretical linguistics is mostly about cognitive representations
- Many CLA research cares about representations
- Formal linguistics & CLT demand careful thought about representations

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What is h?: A way forward

- CLT stands to direct CLA more towards representations
- CLA and CLT add the dimension of learnability to theoretical linguistics



The obvious answer

$$h(L(g_t)) = g_h, g_t = g_h$$

A learner whose input was generated by a grammar gt should acquire grammar gh

• Learning succeeds when the grammar that the child hypothesizes is "the same" as the target grammar

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A learner whose input was generated by a grammar gt should acquire grammar gh

- Learning succeeds when the grammar that the child hypothesizes is "the same" as the target grammar
- Reasonable, practical, works under many definitions of "the same."
- **X** Makes several unrealistic assumptions

• Does "sameness" between g_1 and g_2 mean extensional equivalence, or a more intensional notion of equivalence?

 $h(L(g_1)) = g_h$, $L(g_t) = L(g_h)$ or $g_t \equiv g_h$?

CLT is well-equipped to deal with notions of equivalence

• Which formal measure of "sameness" should be used?

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- Input is not really drawn from a single target grammar, but rather a mix

 $h(L(g_1) \cup L(g_2)... \cup L(g_n)) = g_h \text{ or } \{g_{h_1}, g_{h_2}, ..., g_{h_m}\}$?

Variation is a ubiquitous fact about the input to CLA From production and perception noise, individual differences, sociolinguistics

Learning from noisy input is a heavily studied concept in CLT

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 - ? Should the learner acquire one mega-grammar or many grammars? Depending on G, what if no single grammar can cover the entire input?

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- Learning from noisy input is a heavily studied concept in CLT
 - ? Should the learner acquire one mega-grammar or many grammars? Depending on G, what if no single grammar can cover the entire input?
 - ? How does this direct our notions of appropriate measures of equivalence?

- Which formal measure of "sameness" should be used?
- Input is not really drawn from a single target grammar, but rather a mix
- Grammars that are not formally equivalent may be practically equivalent

 $h(L(g_1)) = g_h, \quad L(g_t) \approx L(g_h) \quad \text{or } g_t \approx g_h ?$

If the differences in their extensions virtually never appear in their outputs or g_1 and g_2 parse an utterance differently in a way that causes no confusion

- Virtually never appear" could be handled with a "close enough" error term
- ✗ Semantic equivalence isn't good enough for "causes no confusion"
 → May be one situation where language-only isn't good enough...

Some recommendations for measuring success

- Characterize learning through noise
- Relax the idea of matching a single target grammar
- → Formalize "close enough" learning

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- Characterize learning through noise
- Relax the idea of matching a single target grammar
- → Formalize "close enough" learning
 - ✓ Consider abductive learning \leftarrow we like it \bigcirc^1
 - Check how "close enough" manifests empirically through studies of language in use and of child development

Learning Trajectories

CLA is Online/Incremental

- As opposed to batch learning
- Intermediate hypotheses/learner states are important
- Much can be inferred about intermediate states from learner behavior Across all levels of the grammar Both observational/corpus and experimental methodologies

 $h(L(g_t)) = \langle g_{h1}, g_{h2}, \dots, g_{hn} \rangle$

The learner should pass through a sequence of hypothesis grammars

Learning Trajectories

Remember French liaison just a bit ago??

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Learning Trajectories

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- Much can be inferred about intermediate states from learner behavior

Consequence: Online learning is most relevant for CLA × CLT. A fully successful CLT perspective should account for intermediate states, not just the final state

The Output of CLA: Summary

The output is a (sequence of) grammar(s), but evaluation is obscure

- The simplest success metric is not necessarily appropriate
- CLA is online and incremental

→ intermediate states should be taken into account

 \rightarrow good news if you like algorithmic thinking! \leftarrow we do \bigcirc

• Learning may be "good enough"

→ abductive learning is a good idea!

→ robustness to input noise (regardless of its origin) is crucial

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The output is a (sequence of) grammar(s), but evaluation is obscure

- The simplest success metric is not necessarily appropriate
- **CLA is of The Big Caveat:**
 - → interm We aren't advocating against abstracting the → good r
 Learning
 problem of CLA. Rather, that it is important to
 sometimes revisit the consequences of abstraction
- - → abduc Too much takes us away from CLA x CLT

→ robustness to input noise (regardless of its origin) is crucial
Conclusions

The Input to CLA: Pros and Cons for CLT Research

The Cons:

- Can't assume conveniently selected data presentations in terms of ordering (e.g., regulars first) or completeness (e.g., full paradigms)
- Must assume finite input that is probably smaller than we'd hope for
- Negative examples would be helpful, but we're out of luck

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The Pros:

- Don't have to prove learning under arbitrary data presentations Often assume long-tailed distributions that are not deliberately adversarial
- Language input (generally streams of symbols) is reasonable to assume 😅

The Output of CLA: Pros and Cons for CLT Research

The Cons:

- Noisy data, non-exact-match evaluation, and the possibility that no single grammar accounts for the entire the input all make life more complicated
- So do incremental learning and caring about learning trajectories
- ...and every piece of the grammar is going to have different requirements

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The Pros:

- Incremental learning is good news if you like algorithmic thinking
- Intermediate states provide a wealth of evidence about the learner Both in terms of the hypothesis space and the learning strategy

A Blatant Non-Conclusion

We.DU.EXCL have made progress towards a formalization of CLA, but we are still far from proposing a gold standard

→ We hope this talk inspires some discussion!

- What other desiderata are there for a formalization of CLA?
 - e.g., Order of acquisition tells us what prior info learners can access The input doesn't contain underlying representations How does adult variation (French liaison again...) reflect imperfect convergence during learning?...
- What other important considerations are there for CLA x CLT?
- Where have we missed the mark?



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Desiderata: The Output

A Concrete Example: Paradigm Saturation¹



¹Chan 2008, Lignos & Yang 2016, Payne 2022, Kodner 2023

A Concrete Example: Paradigm Saturation

UD Finnish PS



A Concrete Example: Paradigm Saturation



A Concrete Example: U-Shaped Learning

English Past Tense Phase 1

- Past tense is marked inconsistently
- But is formed accurately when used

Phase 2 The U-Shape

- Past tense is marked consistently
- But irregulars are sometimes regularized
- Error rate declines over time

Interpretation: Phase 1 \rightarrow Phase 2 **Indicates a new hypothesis** $g_{h1} \rightarrow g_{h2}$



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